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The Prediction of the Tensile Properties of Flexible Foams

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Synopsis

An idealized model for flexible foam in tension is developed. The model includes large deformations and cell structure orientation before and after deformation. It is predicted that two quantities affect the initial modulus of an unoriented foam: the density and a cell structure parameter. Data for latex foam show that the model correctly predicts the initial tensile modulus. The model also predicts that cell structure orientation is a reasonable method for achieving a desired modulus without altering the density.

INTRODUCTION

Flexible foams are used primarily for cushioning. There are several reasons for studying their mechanical properties in tension. First, it is commonly accepted that the small strain modulus in tension and compression are identical,[1,2] although there is no paper devoted to this subject. Rusch[2] claims that the tensile modulus is one of the four phenomenological parameters that may be used to describe a foam's compression response. Another justification is that a cushion which is compressed in one direction will be in tension in another due to the nonuniform character of the loading. An indentor which locally compresses a cushion causes a significant tensile strain along its top at the edges of the indentor. The net indenting force is a function of the compression and tension properties of the foam.

Models for the tensile properties of foam were developed by Gent and Thomas[3] and Harding.[4] These authors treat foam as a structure consisting of elastic bars that intersect at points containing rigid material. They conclude that the tensile modulus is a function of the foam density, solid-phase density, and solid-phase modulus and that there is no dependence upon a cell structure parameter. This author concludes that a cell structure parameter which significantly influences the tensile modulus exists. Furthermore, it is shown that initial cell structure orientation is another factor which controls the modulus.

DEVELOPMENT OF MODEL

Any general mathematical model of latex foam should consider these three factors:

1. Foam consists of large amorphous masses of material which connect strands of rubber in circular patterns. The appearance is that of an interconnecting network with the globules at the junction points.
2. During elongation, the fibers stretch and orient in the direction of stretching. The globules remain relatively undeformed and thereby increase the local strain of the fibers.
3. Orientation of the structure prior to loading is possible.

In Gent and Thomas,[3] the first is considered in predicting the small strain modulus. However, the analysis is limited to small strains and cannot be extended to finite strain phenomena such as tearing or to oriented foams. The presently proposed model consists of fibers of any assumed orientation in the unstrained state. The fibers have the properties of the rubber phase of the foam. Their ends connect into rigid spheres, or "dead volumes." For simplicity, we assume that each fiber has an average size specified by a length l and an area a and that a fixed number of fibers n connect to each sphere of diameter D.

Now consider an elemental plan of area dA and unit normal μ at a point within the foam, as depicted in Figure 1. We want to consider the traction exerted on the material on one side of dA by the material on the other. Let $\mathbf{s}$ be a unit vector of arbitrary direction. Associate with $\mathbf{s}$ a solid angle element $d\omega$ defined by a complete rotation of the vector $\mathbf{s} + d\mathbf{s}$ about $\mathbf{s}$. Next we introduce a direction distribution function p for the bar elements, so that the integral of p over all spatial directions, i.e., over a solid angle 2π, is unity. Then $\kappa p d\omega$ denotes the number of bars per unit volume having directions bounded by $d\omega$. The number of bars (N_B) crossing dA and having directions bounded by $d\omega$ is given by

$$N_B = \kappa p l \mathbf{s}_\mu dA d\omega \tag{1}$$

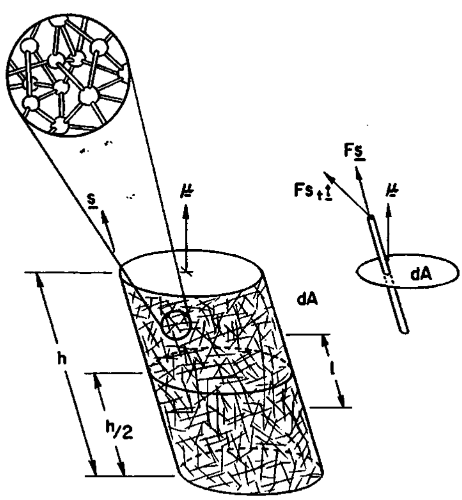

Fig. 1. Differential volume of material of arbitrary size.

where l is a length in the direction of $\mathbf{s}$ which together with dA bounds a cylindrical volume above which no new elements contribute to the sum in eq. (1) and below which elements are neglected in arriving at this sum. By intuition, l is comparable to the average length of an element and is chosen as such.

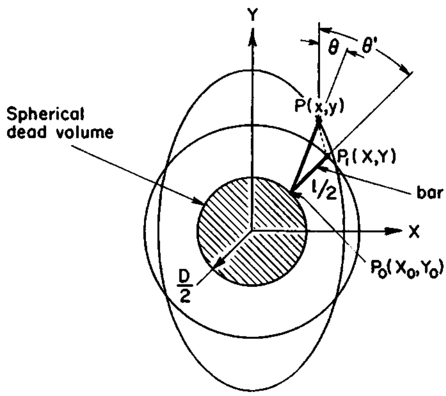

Fig. 2. Deformation of a single bar element. Deformation is such that element consisting of bar and "dead volume" deforms according to the global deformation of the specimen, namely, in a uniaxial tension test: $x = \lambda_t X$, $y = \lambda Y$, $x - X_0 = (\lambda_t(1 + \beta) - \beta)X_0/\beta$, and $y = Y_0 = (\lambda(1 + \beta) - \beta)Y_0/\beta$.

The effect on the tractive force of a spherical element crossing dA is to transmit the stresses from the bars connected to that element. This means that the effective number of bars intersecting dA is greater than N_B. If n bars connect to one sphere and each bar to two spheres, then there are $(2/n)\kappa$ spheres per unit volume. If $N_{\mathbf{s}}$ is the number of spheres intersecting dA having bars in the $\mathbf{s}$ direction, then

$$N_{\mathbf{s}} = (2/n)\kappa p D \mathbf{s}_\mu dA d\omega. \tag{2}$$

The number of bars within the solid angle $d\omega$ and intersecting spheres on dA is

$$N_{BS} = \frac{n}{2} N_{\mathbf{s}} = \kappa p D \mathbf{s}_\mu dA d\omega. \tag{3}$$

The total number of bars in the $\mathbf{s}$ direction crossing dA is $N_B + N_{BS}$.

The force component in the $\mathbf{r}$ direction on a fiber in the $\mathbf{s}$ direction is

$$F_{\mathbf{r}} = F\mathbf{s}\cdot\mathbf{r} = F\mathbf{s}_{\mathbf{r}} \tag{4}$$

and the traction in the $\mathbf{r}$ direction due to all elements in the $\mathbf{s}$ direction follows as

$$dT_{\mathbf{r}}^{\mu} = \frac{F_{\mathbf{r}}(N_B + N_{BS})}{dA} = Kla\left(1 + \frac{D}{l}\right)\mathbf{s}_\mu \mathbf{s}_{\mathbf{r}} d\omega. \tag{5}$$

The total traction on dA due to all the elements in all the directions is determined by integrating eq. (5) over $\Omega = 2\pi$, which is the solid angle covering these directions, that is to say,

$$T_r^{\mu} = \kappa la(1 + \beta) \int_{2\pi} f s_{\mu} s_r p d\omega \tag{6}$$

where $\beta = D/l$, and $f = F/a$ is the tensile stress in a bar element.

The fiber stress f is assumed to be given by the stress in a solid rubber bar which is subjected to a stretch ratio λ_1. In general, λ_1 is the stretch ratio in the direction given by $\mathbf{s}$ and is related to the global, or macroscopic, strain through a second-order transformation. For our purposes, we need only consider a uniaxial tension test where the global stretch ratios in the axial and transverse direction are λ and λ_t, respectively. In Figure 2, a particle is shown along with one of the bars associated with that particle. Since each bar is connected to two spheres, only half a bar is drawn. Points P_0 and P_1 are the endpoint positions of the undeformed bar and P is the deformed position of the particle initially at P_1. It is assumed that this combined bar and sphere element is subjected to the global stretch ratios λ and λ_t, that is,

$$x = \lambda_t X \qquad y = \lambda Y. \tag{7}$$

Furthermore, by similar triangles,

$$\frac{x}{X_0} = \frac{y}{Y_0} = \frac{l + D}{D} = \frac{1}{\beta} + 1. \tag{8}$$

From the definition of λ_1 we obtain

$$\begin{aligned} \lambda_1^2 &= \frac{(x - X_0)^2 + (y - Y_0)^2}{(X - X_0)^2 + (Y - Y_0)^2} \\ &= (\lambda_t(1 + \beta) - \beta)^2 \sin^2\theta' + (\lambda(1 + \beta) - \beta)^2 \cos^2\theta' \end{aligned} \tag{9}$$

where θ' is defined in Figure 2. The angle θ in the deformed material is related to θ' in the undeformed material by

$$\tan\theta = \frac{x - X_0}{y - Y_0} = \frac{\lambda_t(1 + \beta) - \beta}{\lambda(1 + \beta) - \beta} \tan\theta'. \tag{10}$$

Substitution of eq. (10) into eq. (9) yields

$$\lambda_1^2 = \frac{(\lambda_t(1 + \beta) - \beta)^2}{1 - \alpha^2 \cos^2\theta} \tag{11}$$

where

$$\alpha^2 = 1 - k^2 = 1 - \frac{[\lambda_t(1 + \beta) - \beta]^2}{[\lambda(1 + \beta) - \beta]^2}. \tag{12}$$

The dead volume increases the local strain and the amount of fiber orientation in the axial direction. The probability of finding a bar within the conical sector bounded by θ in the deformed body is a measure of the orientation. This equals the probability of finding one within the conical sector bounded by θ' in the undeformed body, or

$$P(\theta_D \leqslant \theta) = P(\theta_u \leqslant \theta'). \tag{13}$$

The probability $P(\theta_u \leqslant \theta')$ is given by the direction distribution function integrated over the solid angle within the conical sector which is bounded by θ', i.e.,

$$P(\theta_u < \theta') = \int_0^{2\pi}\int_0^{\theta'} p_u(\alpha') \sin\alpha \, d\alpha' d\gamma' = \int_0^{2\pi}\int_0^{\theta} p(\alpha) \sin\alpha \, d\alpha d\gamma \tag{14}$$

where p_u is the direction distribution function in the undeformed body.[5] Taking the derivative with respect to θ yields

$$\begin{aligned} p(\theta)\sin\theta &= p_u(\theta'(\theta))\sin\theta' \frac{d\theta'}{d\theta} \\ &= \frac{p_u(\theta'\theta)k}{1 - \alpha^2\cos^2\theta} \end{aligned} \tag{15}$$

where $\theta'(\theta)$ is evaluated using eq. (10).

To complete the derivation we must express κla in terms of more easily measured quantities. If ρ is the density of solid rubber and ρ_f is that of foam, then

$$\rho_f = \rho\kappa\left(la + \frac{\pi D^3}{3n}\right) = \kappa la\rho\left(1 + \frac{\beta}{3}\frac{\pi D^2}{na}\right)$$

or

$$\kappa la = \frac{\rho_f}{\rho}\frac{1}{\dfrac{na}{\pi D^2} + \dfrac{\beta}{3}}\frac{na}{\pi D^2}. \tag{16}$$

Substituting eqs. (15) and (16) into eq. (6) evaluated in the axial and transverse direction yields

$$\sigma = 2\pi \frac{1+\beta}{\dfrac{na}{\pi D^2} + \dfrac{\beta}{3}} \frac{na}{\pi D^2}\frac{\rho_f}{\rho} k \int_0^{\pi/2} \frac{f(\lambda_1)p_u(\theta')\cos^2\theta\sin\theta}{1 - \alpha^2\cos^2\theta} d\theta \tag{17a}$$

$$\sigma_t = 2\pi \frac{1+\beta}{\dfrac{na}{\pi D^2} + \dfrac{\beta}{3}} \frac{na}{\pi D^2}\frac{\rho_f}{\rho} k \int_0^{\pi/2} \frac{f(\lambda_1)p_u(\theta')\sin^3\theta}{1 - \alpha^2\cos^2\theta} d\theta = 0 \tag{17b}$$

where θ' and λ_1 are known functions of θ given by eqs. (10) and (11), respectively.

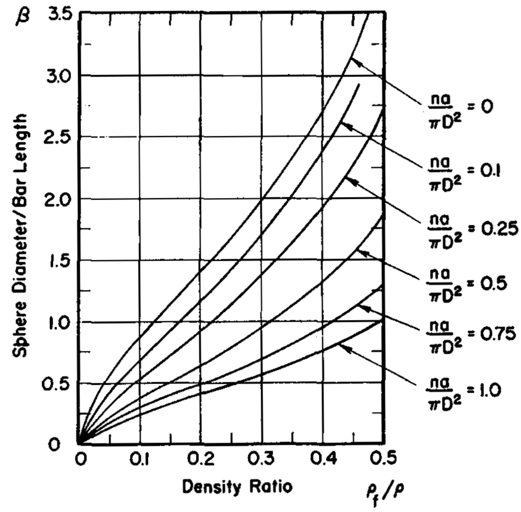

Fig. 3. Sphere diameter/bar length vs. density ratio, β vs. ρ_f/ρ.

By considering a spherical volume of diameter $D + l$ centered at the middle of a dead volume, one obtains

$$\rho_f \frac{\pi}{6} (D + l)^3 = \rho \left(n \frac{l}{2} a + \frac{\pi}{6} D^3 \right)$$

or

$$\frac{\rho_f}{\rho} = \frac{3\beta^2}{(1 + \beta)^2} \left(\frac{na}{\pi D^2} + \frac{\beta}{3} \right). \tag{18}$$

This equation is plotted in Figure 3 using $na/\pi D^2$ as a parameter. Since na is the total area of the bars connected to a dead volume and πD^2 is the surface area of the dead volume, then their ratio, $na/\pi D^2$, is a measure of the amount of spherical area covered by bar elements. This quantity is affected by the bubble size as a result of whipping and the latex rheological properties during gelling.[7]

Thus, for a given rubber phase stress–strain law, a specified direction distribution function of undeformed fibers, a known foam density, and a value of $na/\pi D^2$, chosen as prescribed in the next section, the stress–strain behavior in tension may be calculated from eqs. (17) and (18). In particular, the small strain tensile modulus will be calculated for foams of various orientations.

YOUNG'S MODULUS AND POISSON'S RATIO

Unoriented Foam

Young's modulus, or the zero strain modulus, is obtained by substituting

$$f(\lambda_1) = E(\lambda_1 - 1), \lambda = 1 + \epsilon, \lambda_t = 1 + \epsilon_t \tag{19}$$

into eqs. (17) and letting ϵ and ϵ_t be small. For this example, an initially unoriented foam is considered in which case $p_u = 1/2\pi$. Equations (17) then reduce to

$$E_f/E = \frac{1}{5}\left(1 - \frac{2}{3}\nu_f\right)\frac{3\beta^2}{1+\beta}\frac{na}{\pi D^2} \quad \nu_f = \frac{1}{4} \tag{20}$$

where E_f is the foam modulus, ν_f is Poisson's ratio, and E is the modulus for the rubber.

Data were obtained for two different foam compounds to check against the values of Young's modulus and Poisson's ratio as predicted by eqs. (20). Both compounds were foamed by mechanical beating and gelled by sodium silicofluoride.[6] The Appendix contains the foam formulations. The results for the first compound are due to Gent and Thomas[3] and for the second, by the author.

TABLE I
Poisson's Ratio for Various Density Foams

Density, (lb/in.³) × 100	ν_f
0.707	0.18
0.396	0.25
0.67	0.15
0.381	0.15
0.418	0.34
0.463	0.34
0.458	0.23
0.530	0.23
0.627	0.23
0.727	0.20
0.749	0.20
0.661	0.28
	$\bar{\nu}_f$ = 0.23

Table I presents data for Poisson's ratio for 12 samples prepared by the author. There is a wide dispersion of values, however, their mean is 0.23, which is within 10% of predicted. This is not a validation of the theory. It is only used to justify letting $\nu_f = 0.25$ in eq. (20) which becomes

$$\frac{E_f}{E} = \frac{1}{6}\frac{3\beta^2}{1+\beta}\frac{na}{\pi D^2} \tag{21}$$

With a specified density ratio, ρ_f/ρ, and a value of $na/\pi D^2$, β can be obtained from eq. (18) or Figure 3. This in turn can be used to evaluate eq. (21). This procedure was followed, and the result is plotted against density ratio in Figure 4 for $na/\pi D^2 = 0.5$ and 1 and in Figure 5 for $na/\pi D^2 =$ 0.15 and 1. The theory of Gent and Thomas[3] corresponds to $na/\pi D^2 = 1$. The curve for $na/\pi D^2 = 0.5$ fits Gent and Thomas's data better, and $na/\pi D^2 = 0.15$ fits this author's data better. The fact that one value of

$na/\pi D^2$ applies for each group of foam samples indicates that this quantity is directly related to processing variables, such as frothing technique and surface properties of the uncured compound. Both of these were held constant for the experiment. Thus, once $na/\pi D^2$ is determined for a few samples, the same value can be used for all subsequent ones prepared by the same process.

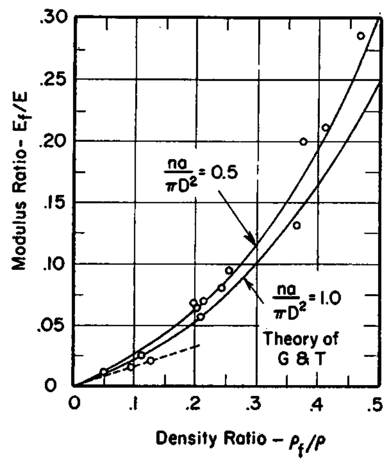

Fig. 4. Modulus vs. density. Data derived by Gent and Thomas[3] for natural rubber latex foam.

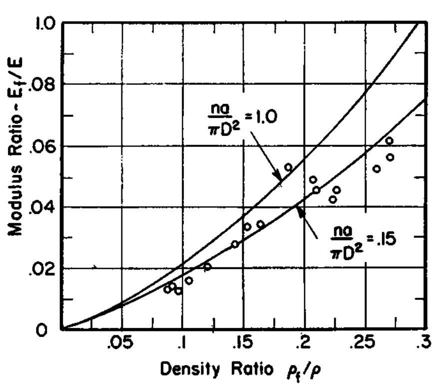

Fig. 5. Modulus vs. density. Data for SBR 2105 latex foam showing predicted and actual foam modulus.

Figure 6 shows the effect of varying $na/\pi D^2$. The modulus is relatively insensitive to structural changes in the region of $na/\pi D^2 = 0.5$. This is the case for all densities since the curve passes through a maximum, independent of density, at this point. We show this by setting the differential of eq. (21) to zero while holding ρ_f constant, i.e., the differential of ρ_f is zero. Mathematically,

$$d\left(\frac{E_f}{E}\right) = \frac{\partial(E_f/E)}{\partial\beta}\,d\beta + \frac{\partial(E_f/E)}{\partial\left(\frac{na}{\pi D^2}\right)}\,d\left(\frac{na}{\pi D^2}\right) = 0 \tag{22a}$$

$$d\left(\frac{\rho_f}{\rho}\right) = \frac{\partial(\rho_f/\rho)}{\partial\beta}\,d\beta + \frac{\partial(\rho_f/\rho)}{\partial\left(\frac{na}{\pi D^2}\right)}\,d\left(\frac{na}{\pi D^2}\right) = 0. \tag{22b}$$

Solving this homogeneous system yields the desired result, namely, $na/\pi D^2 = 0.5$ for optimum foam modulus at a specified density. The flatness of the curve at the maximum is fortuitous since it allows for processing variations without much loss in modulus.

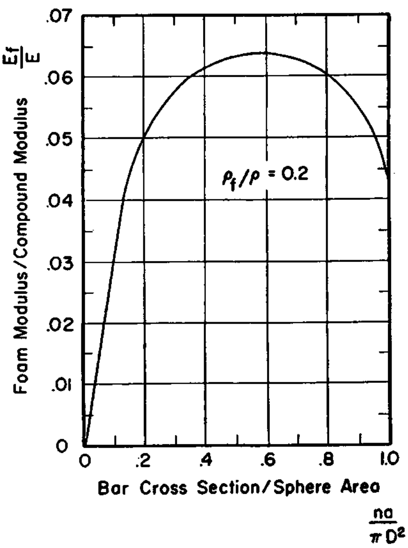

Fig. 6. Foam modulus plotted as a function of structure for a constant density.

Oriented Foam

There are two limiting cases in an oriented foam: (a) complete alignment of the bars with the loading direction and (b) complete alignment normal to the loading direction. In the first case,

$$\begin{aligned} p_u(\theta') &\to \infty \text{ as } \theta' \to 0 \\ p_u(\theta') &= 0 \qquad \theta' \neq 0 \end{aligned} \qquad \int_0^{\pi/2} p_u(\alpha')\sin\alpha'\,d\alpha' = \frac{1}{2\pi}. \tag{23}$$

Upon substitution into eq. (17a) and after assuming small strains, one obtains the modulus, that is to say

$$\frac{E_f}{E} = \frac{3\beta^2}{1+\beta}\,\frac{na}{\pi D^2}. \tag{24}$$

This is six times the value for an unoriented material. The second case yields the physically meaningless result that the initial modulus is zero.

An intermediate possibility of industrial interest is a partially oriented foam. Such a condition could result from mold effects or from a purposeful orientation during gelling. An exact knowledge of the direction distribution function cannot normally be known. To show the effects, we assume a function of the form

$$p_u(\theta') = \frac{2}{\pi^2}\sin\theta'. \tag{25}$$

This function describes a foam with more bars oriented in the transverse than in the axial direction. Substituting this distribution into eqs. (17) yields

$$\frac{E_f}{E} = \frac{1}{10}\frac{3\beta^2}{1+\beta}\frac{na}{\pi D^2} \qquad \nu_f = \frac{1}{5} \tag{26}$$

which is 0.6 times the unoriented foam modulus.

The consequences of this softening extends to compression. The tensile modulus, it is claimed,[2] equals the initial compression modulus of a foam. During use, latex foam will compression set, lose height, and soften without any visible degradation of the foam's rubber phase. The orientation of the bar elements can explain this phenomenon. As another indication of the effect of orientation has been given by Zocco,[8] who used orientation during gelling to alter the compressive stiffness of a polyurethane foam.

SUMMARY

A mathematical model for the prediction of the tensile properties of a flexible foam is derived. The model consists of randomly interspersed bars and spheres forming a network of any predetermined orientation. Two parameters are introduced, the ratio of the bar cross section to sphere area and the ratio of the bar length to sphere diameter. When the former ratio is unity and the foam is unoriented, this model reduces to that of Gent and Thomas.[1] It was found that this model matches the experimental data better than any of the previous ones.

For unoriented foam, it is predicted that the optimum stiffness is achieved when $na/\pi D^2 = 0.5$. The stiffness can also be controlled by orienting the bars, e.g., decreased by aligning them normal to the loading direction. Data on compression set and softening of latex foam substantiate this result.

APPENDIX

Foam Formulations by Weight

Component	Dry, phr
Gent and Thomas[3]	
NR Latex, 60%	100
10% Potassium ricinoleate	7
5% Cetyl trimethylammonium bromide	3
20% Potassium chloride	2.5
50% Sulfur paste	2.5
50% Zinc oxide paste	3
50% Zinc diethyl dithiocarbamate	1
50% Zinc 2-mercaptobenzothiazole	0.3
50% sym-Di-β-naphthyl-*p*-phenylene diamine	0.5
25% Sodium silicofluoride	1–1.5
Author's Foam	
SBR 2105 latex	100
20% Potassium oleate	1.5
62% Sulfur paste	3
55% Zinc oxide paste	6
50% Ethazate[a] (zinc diethyl dithiocarbamate)	0.75
Trimene Base[a] (reaction product of ethyl chloride, formaldehyde, and ammonia)	0.75
OXAF (zinc salt of 2-mercaptobenzothiazole)	1.5
14% Ammonium hydroxide	4 (wet)
Talc	40
Antioxidant	1 (wet)
Sodium silicofluoride	3

[a] Uniroyal, Inc., trademark.

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